Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level In Mechanics 3

Paper WME03/01

Question Number	Scheme	Marks
1(a)	$V = \pi \int_1^a y^2 dx$	
	$V = (\pi) \int_1^a \frac{1}{x^2} dx = (\pi) \left[-\frac{1}{x}\right]_1^a$	M1A1
	$V = \pi \left(1 - \frac{1}{a}\right) *$	A1*
		(3)
(b)	$(\pi) \int xy^2 dx$	M1
	$(\pi) \int_1^a \frac{1}{x} dx = (\pi)[\ln x]_1^a = (\pi) \ln a$	dM1A1
	$(\pi)\left(1 - \frac{1}{a}\right)\overline{x} = (\pi) \ln a$	
	$\overline{x} = \frac{a \ln a}{a - 1}$	M1A1
		(5)
		[8]

(a)

M1 Use of $\int y^2 dx$ AND an attempt at algebraic integration (power increasing by one)

A1 Correct integration. π not needed.

A1* Given result reached from fully correct working. If π not included from the start, its inclusion must now be justified.

(b)

M1 Use of $\int xy^2 dx$, must have substituted for *y*. π not needed.

dM1 Attempt at algebraic integration (*ln x* neds to be seen)

A1 Correct result after substitution of limits. π not needed.

M1 Use of $\frac{\int xy^2 dx}{\int y^2 dx}$. If π and/or ρ appear, they must appear consistently.

A1 Correct final answer. They lose this mark if they leave $1-\frac{1}{a}$ in the denominator.

Question Number	Scheme	Marks
2(a)	$F = \dfrac{k}{(x+R)^2}$	M1
	$x = 0, F = mg \to mg = \dfrac{k}{R^2}$	M1
	$k = mgR^2 \to F = \frac{mgR^2}{(x+R)^2}$*	A1*
		(3)
(b)		
	$mv\dfrac{dv}{dx} = -\dfrac{mgR^2}{(x+R)^2} \quad or \quad m\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -\dfrac{mgR^2}{(x+R)^2}$	M1
	$\dfrac{1}{2}v^2 = -\int \dfrac{gR^2}{(x+R)^2}dx$	dM1
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x+R}(+c)$	A1
	$x = R, v = U$	M1
	$\dfrac{U^2}{2} = \dfrac{gR^2}{2R} + c \to c = \dfrac{U^2 - gR}{2}$	A1
	$x = 0 \to \dfrac{1}{2}v^2 = gR + \dfrac{U^2 - gR}{2}$	
	$v^2 = U^2 + gR \to v = \sqrt{U^2 + gR}$	M1, A1
		(7)
		[10]
ALT1 (b)	$\dfrac{mv^2}{2} - \dfrac{mU^2}{2} = -m\int_R^0 \dfrac{gR^2}{(x+R)^2}\ dx$	M1
	$\dfrac{v^2}{2} - \dfrac{U^2}{2} = \left[\dfrac{gR^2}{x+R}\right]_R^0$	dM1 A1
	$\dfrac{v^2}{2} - \dfrac{U^2}{2} = \dfrac{gR^2}{R} - \dfrac{gR^2}{2R}$	M1 A1
	$v^2 = U^2 + gR \to v = \sqrt{U^2 + gR}$	M1, A1

Question Number	Scheme	Marks
ALT2 (b)	$mv\frac{dv}{dx} = -\frac{mgR^2}{(x+R)^2}$	M1
	$\int_U^V v dv = -\int_R^0 \frac{gR^2}{(x+R)^2}\ dx$	dM1
	$\left[\frac{v^2}{2}\right]_U^v = \left[\frac{gR^2}{x+R}\right]_R^0$	A1
	$\frac{v^2}{2} - \frac{U^2}{2} = \frac{gR^2}{R} - \frac{gR^2}{2R}$	M1 A1
	$v^2 = U^2 + gR \to v = \sqrt{U^2 + gR}$	M1, A1
		(7)

(a)

M1 Setting up an inverse square relationship between F and $(x+R)$ .Can be negative

Allow with $d = x + R$ or $k = GMm$

dM1 Clear use of $x = 0$ and $F = mg$ to find value of constant (k or GM)

A1* Given result reached with both M marks clearly earned. Must be positive

(b)

M1 Use of $mv\frac{dv}{dx}$ or $m\frac{d}{dx}(\frac{1}{2}v^2)$ to form equation. Condone sign error.

dM1 Separate variables to produce form ready for integration. Condone sign error.

A1 Correct integration. Sign must be correct now. Constant of integration not needed.

M1 Use of initial conditions in the result of an integration to find constant.

A1 Correct value for their c (for the side they place c on).

M1 Finding a value for v (or v^2) using $x = 0$. v^2 must come from a dimensionally correct expression.

A1 Correct expression for v.

ALT1 (b) uses the change in KE = work done

M1 Equates change in KE to the integral of F . Condone sign error

dM1 Integrates (power of $(x+R)$ must increase). Condone sign errors. Limits not needed.

A1 Correct integration. Sign must be correct now for their LHS. Limits not needed.

M1 Substitution of both limits R and 0 into definite integration.

A1 Correct limits, the correct way round for their equation.

Final two marks are the same as the main scheme.

ALT2 (b) uses definite integration

M1 Use of $mv\frac{dv}{dx}$ to form equation. Condone sign error.

dM1 Separate variables to produce form ready for integration. Condone sign error. Limits not needed.

A1 Correct integration. Sign must be correct now. Limits not needed.

M1 Substitution of both limits in definite integration. Must be 0, R, v and U

A1 Correct limits, the correct way round.

Final two marks are the same as the main scheme.

S.C. If they redefine x as the distance from the centre of the earth for (b) and use limits R and $2R$ correctly, full marks can still be gained in (b)

Question Number	Scheme	Marks
3(a)	$cos\,\theta = \frac{4}{5}, sin\,\theta = \frac{3}{5}$	B1
	$\omega = \pi$	B1
	$T_A\,cos\,\theta - T_B\,cos\,\theta = 600g$	M1A1
	$(T_A - T_B = 750g)$	
	$T_A\,sin\,\theta + T_B\,sin\,\theta = 600 \times \omega^2 \times (5\,sin\,\theta)$	M1A1
	$(T_A + T_B = 3000\pi^2)$	
	Solve their two equations simultaneously	dM1
	$T_A = 1500\pi^2 + 375g = 18000(N)$, $18500(N)$, 18kN, 18.5kN	A1
	$T_B = 1500\pi^2 - 375g = 11000(N)$, $11100(N)$, 11kN, 11.1kN	A1
		(9)
(b)	If the length of the arms increased, then the radius of the circle would increase.	B1
	Therefore the total tension would increase.	dB1
		(2)
		[11]

(a)

B1 Correct trig **used** anywhere.

B1 Angular speed **seen**.

M1 Attempt at vertical resolution. Allow mass as m

A1 Correct equation in $\theta. m = 600$ needs to be used now or later.

M1 Attempt at horizontal resolution. Acceleration in either form. Attempt at R not needed. Condone sin/cos confusion and use of the same angle for both forces. Allow mass as m

A1 Correct equation in w and θ . m and R must be substituted (= 3 or 5sin θ may be seen later on)

dM1 Solve their equations to find at least one tension. Dependent on both previous M marks.

A1 Correct T_A (must be 2/3 s.f.)

A1 Correct T_B (must be 2/3 s.f.) (only penalise over accuracy on T_A if in both)

(b)

B1 Correct statement about the effect on the radius of the motion.

dB1 Conclusion that **total tension** would be greater (must reference the total tension) following a correct statement about the radius.

Question Number	Scheme					Marks
4(a)		Top cone	inside cone	C	S	
	Mass ratio	(-) 1	(-) 1	8	6	B1
	y distance	$5a$	$3a$	$2a$	$\bar{y}$	B1
	My	$5a$	$3a$	$16a$	$6\bar{y}$	
	$8 \times 2a - 1 \times 5a - 1 \times 3a = 6\bar{y}$					M1A1 **ft**
	$6\bar{y} = 8a \rightarrow \bar{y} = \frac{4}{3}a$					A1
						(5)
(b)	$tan\, \alpha = \frac{3}{8}$ $(\alpha = 20.556 \ldots.\, or\ 69.44 \ldots.)$					B1
	$tan\, \beta = \frac{\frac{3a}{2}}{4a - \frac{4a}{3}} = \frac{9}{16}$ $(\beta = 29.357 \ldots or\ 60.642 \ldots.)$					M1A1ft
	$\alpha + \beta = \theta = 50^o$ (or better 49.91379…)					A1
	$or\ 180 - 69.44 \ldots - 60.64 \ldots. = 50°$					**(4)**
						[9]
ALT (b)	$\cos\theta = \frac{AB^2 + BG^2 - AG^2}{2 \times AB \times BG}$					B1
	$BG^2 = (1.5a)^2 + (4a - \bar{y})^2 \quad (= \frac{337a^2}{36})$ $AG^2 = (3a)^2 + (\bar{y})^2 \quad (= \frac{97a^2}{9})$ $\{AB^2 = (1.5a)^2 + (4a)^2 \quad (= \frac{73a^2}{4})\}$					M1
	$\cos\theta = \ldots\ldots\ldots\ldots\ldots\ldots = 0.6439 \ldots \ldots$					A1ft
	$\theta = 50^o$ (or better 49.91379…)					A1

(a)

B1 Correct mass ratio seen for 3 cones and *S*. Allow consistent (-)

B1 Correct distances for the 3 cones. (Allow distances from vertex (3*a*, 5*a*, 6*a*) or small plane face (-*a*, *a*, 2*a*).) Condone missing (-)

M1 Dimensionally correct moments equation about any parallel axis. Must include 4 terms.

A1ft Correct moments equation follow through their distances.

A1 $\bar{y} = \frac{4}{3}a$ o.e.

SC if a's are missing B1B0M1A1ftA0 is the maximum available

(b)

B1 Correct expression for $tan\,\alpha$ or α seen (either way round)

M1 Correct attempt to use their $\bar{y}$ to find $tan\,\beta$ (either way round)

A1ft Correct expression for $tan\,\beta$ or β (either way round). Ft their $\bar{y}$

A1 50^o or better (0.87 rad or better 0.87116……)

ALT (b) uses the cosine rule with triangle *ABG*

B1 Correct expression for cos θ in terms of *AB*, *AG* and *BG*

M1 Correct attempt to use their $\bar{y}$ to find *AG* and *BG*

A1ft Correct expression for cos θ. Ft their $\bar{y}$

A1 50^o or better

Question Number	Scheme	Marks
5(a)	$\frac{2mge_1}{2a}$ or $\frac{6mg(4a-e_1)}{4a}$	B1
	$mg+\frac{2mge_1}{2a}=\frac{6mg(4a-e_1)}{4a}$	M1A1
	Solve to find either extension	dM1
	$e_1=2a$ and $e_2=4a-e_1=2a$*	A1*
		(5)
ALT (a)	$mg+\frac{2mge_1}{2a}=\frac{6mge_2}{4a}$, $e_1+e_2=4a$	M1A1
	Solve simultaneously to find either extension	dM1
	$e_1=2a$ and $e_2=4a-e_1=2a$*	A1*
(b)		
	$mg+\frac{2mg(2a-x)}{2a}-\frac{6mg(2a+x)}{4a}=m\ddot{x}$	M1A1A1
	$\ddot{x}=-\frac{5g}{2a}x$ $\quad\therefore$ SHM	A1
		(4)
(c)	$\omega^2=\frac{5g}{2a}$	B1ft
	$v^2=\frac{5g}{2a}\left(a^2-\left(\frac{a}{2}\right)^2\right)$	M1A1
	$v=\sqrt{\frac{15ga}{8}}=\frac{\sqrt{30ga}}{4}$	A1 cso
		(4)
ALT (c)	$\frac{2mga^2}{4a}$ or $\frac{6mg(3a)^2}{8a}$ or $\frac{2mg(\frac{3a}{2})^2}{4a}$ or $\frac{6mg(\frac{5a}{2})^2}{8a}$	B1
	$\frac{2mga^2}{4a}+\frac{6mg(3a)^2}{8a}=\frac{2mg(\frac{3a}{2})^2}{4a}+\frac{6mg(\frac{5a}{2})^2}{8a}+\frac{mga}{2}+\frac{mv^2}{2}$	M1A1
	$v=\sqrt{\frac{15ga}{8}}$	A1
		(4)
		[13]

(a)

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically, with two variable tensions and weight (M0 for setting both extensions as e)

A1 Correct equation.

dM1 Solve to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

(b)

M1 Vertical equation of motion with two different variable tensions, weight and $m\ddot{x}$ (allow ma)

A1 Equation with at most one error (allow ma for this mark, which does not count as an error).

A1 Fully correct equation. Must now be $m\ddot{x}$

A1 $\ddot{x} = -\frac{5g}{2a}x$ $\therefore$ SHM. **Must have concluding statement.**

(c)

B1ft Use of their ω^2

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Follow through their ω. Needs amplitude a and $x = \frac{1}{2}a$

A1 Correct equation. No follow through now.

A1 cso

ALT (a) using simultaneous equations

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically with two tensions in e_1 and e_2 and weight AND give a second equation for $e_1 + e_2$

A1 Both equations correct.

dM1 Solves both equations simultaneously to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

ALT (c)

B1 Use of correct EPE

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Allow with $EPE = k\frac{\lambda x^2}{l}$. Must have all terms.

A1 Correct equation.

A1 Correct final answer

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 + mg(2a) = \frac{1}{2}m\left(3\sqrt{ag}\right)^2 - mg(2a\cos 60^o)$	M1A1A1
	$(v^2 = 3ag)$	
	$T + mg = \dfrac{mv^2}{2a}$	M1A1
	$T = \dfrac{m(3ag)}{2a} - mg = \dfrac{mg}{2}$	dM1A1
	$T > 0$, therefore string remains taut and particle performs complete vertical circles.	A1
		(8)
(b)	From initial: $\frac{1}{2}mV^2 = \frac{1}{2}m\left(3\sqrt{ag}\right)^2 + mg(2a - 2a\cos 60^o)$	M1A1
	Or from top: $\frac{1}{2}mV^2 = \frac{1}{2}m(3ag) + mg(4a)$	
	$(V^2 = 11ag)$	
	$T - mg = \dfrac{m(11ag)}{2a}$	M1A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1
		(5)
ALT		**[13]**
(a)	$\dfrac{1}{2}mv^2 + mg(2a\cos 60^o - 2a\cos\theta) = \dfrac{1}{2}m\left(3\sqrt{ag}\right)^2 \rightarrow v^2 = ag(7 + 4\cos\theta)$	M1A1A1
	$T - mg\cos\theta = \dfrac{mv^2}{2a}$	M1A1
	$T - mg\cos\theta = \frac{mag}{2a}(7 + 4\cos\theta)$ AND $\theta = \pi$ or $\cos\theta \geq -1$	dM1
	$T + mg = \frac{mg}{2}(3) \rightarrow T = \frac{mg}{2}$	A1
	$T > 0$, string stays taut and particle completes vertical circles.	A1
(b)	$\theta = 2\pi \rightarrow T - mg\cos 2\pi = \frac{mg}{2}(7 + 4\cos 2\pi)$	M1, M1
	$T - mg = \dfrac{m(11ag)}{2a}$	A1, A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1

(a)

M1 Energy equation from projection to top of the circle. Must have 2 KE terms and a difference in GPE.

A1 Equation with at most one error.

A1 Fully correct equation.

M1 Equation of motion towards centre of circle at top. Allow acceleration in either form.

A1 Correct equation. Acceleration must be in form $\frac{v^2}{r}$. Condone $2a = r$ if substituted later

dM1 Eliminate *v* to form equation for *T*. Dependent on the first two M marks

A1 Correct unsimplified equation for *T*.

A1 Correct inequality with a concluding statement.

SC uses $T > 0$ without $T =$ For the last five marks:

M1 Finds resultant force at the top

A1 $F = \frac{m(3ag)}{2a} = \frac{3mg}{2}$

dM1 Compares their resultant force to the weight. Dependent on the first two M marks

A1 $\frac{3mg}{2} - mg > 0$ or $\frac{3mg}{2} > mg$

A1 Correct concluding statement that must include mention of their being tension in the string at the top

(b)

M1 Attempt energy equation from either initial position, or top, to the bottom of the circle.

A1 Correct equation.

M1 Equation of motion at the bottom of the circle. Allow in terms of *V*.

A1 Correct equation for tension, with *V* eliminated. Must have attempted to calculate *V*

A1 Correct tension and concluding statement.

ALT (a and b) using a general point on the circle

M1 Energy equation from point of projection to a general point. Must have 2 KE terms and a difference in GPE one of which is in θ

A1 Equation with at most one error.

A1 Fully correct equation.

M1 Equation of motion towards centre of circle at the general point. Allow acceleration in either form.

A1 Correct equation. Acceleration must be in form $\frac{v^2}{r}$. Condone $2a = r$ if substituted later

dM1 Eliminate v to form equation for T in m, a, g , θ AND set θ or $\cos\theta$ to evaluate T at the top. Dependent on the first two M marks.

A1 Correct unsimplified equation for T with θ now substituted.

A1 Correct inequality with a concluding statement.

(b)

M1, M1 Substitute for θ or $\cos\theta$ to evaluate T at the bottom

A1, A1 Correct unsimplified equation for T

A1 Correct tension and concluding statement.

Question Number	Scheme	Marks
7(a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7}\left(= \frac{3g}{14} = 2.1\right)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5g(x+2)\sin\theta + F_r(x+2) + \frac{3x^2}{2 \times 2}$	M1A1A1
	$64 = 14(x+2) + 3x^2$	
	$3x^2 + 14x - 36 = 0$	M1
	$x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
(b)	$T = \frac{3 \times 1.84}{2}(= 2.76)$	B1ft
	$0.5a = 2.76 + 1.4 - 2.1 \quad (= 2.06)$ Acceleration down slope, so particle does not remain at *A*.	M1A1
		(3)
		[11]

ALT (a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7}(= 2.1)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5gd\sin\theta + F_r d + \frac{3(d-2)^2}{2 \times 2}$	M1A1A1
	$64 = 14d + 3(d-2)^2$	
	$3d^2 + 2d - 52 = 0$	M1
	$d = 3.8 \rightarrow x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
ALT (b)	$T = \frac{3 \times 1.84}{2}(= 2.76)$	B1ft
	Upslope: $F_{max}(= 2.1)$, Downslope: $0.5g\sin\theta + T\ (= 4.16)$	M1
	4.16 > 2.1 so there is a resultant force down slope, so the particle does not remain at *A*	A1 **(3)**

(a)

B1 Initial speed seen.

B1 Maximum friction seen/used. Award if only seen in (b).

M1 Energy equation with KE, GPE, EPE and WD. If they split the motion up to find the speed when the string begins to extend (= 6 ms^{-1}) only award this mark once they have the equation containing EPE. Allow with $EPE = k\frac{\lambda x^2}{l}$.

A1 Equation with at most one error.

A1 Fully correct equation.

M1 Produce a 3 term in x or d equalling zero (see ALT). This is independent.

M1 Solve a 3 term quadratic to find the extension or distance travelled.

A1 Correct extension. Must be 2 or 3 s.f.

(b)

B1ft Correct expression for the tension at A ft their extension.

M1 Consider the three forces parallel to the plane.

A1 Correct conclusion from comparison of the three forces. Correct working with numerical values seen. Could be an acceleration or correct statement about the forces up/down the slope.